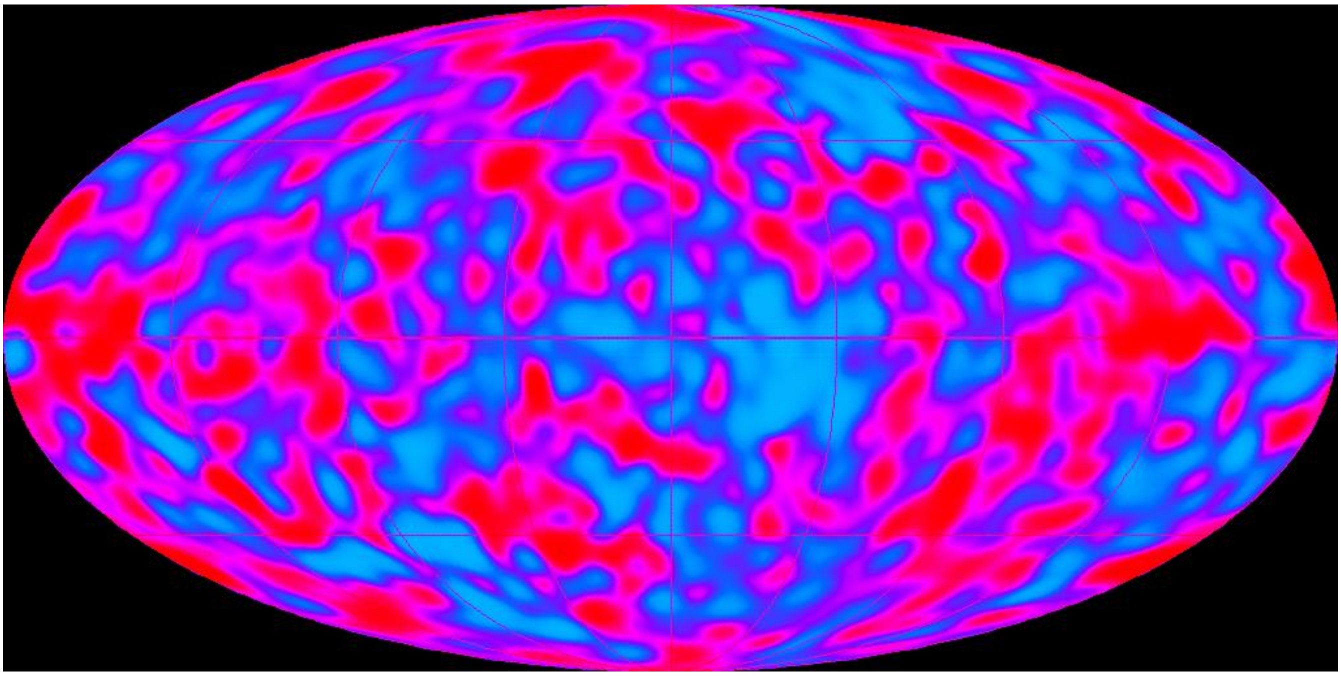
Spectroscopic Determination of Cosmological Distances and Parameters



Version 2.0

Richard Walker 03/2022

Content

Cover picture:

Recording of the cosmic microwave background (CMB) by the COBE satellite in 1992; this relict dates from a period of about 380,000 years after the "Big Bang" and thus appearing extremely redshifted. So today we can just detect it by radio astronomic means. Nobel laureate George Smoot describes this recording as "Looking at the face of God".

Picture: NASA https://de.wikipedia.org/wiki/Hintergrundstrahlung

Supplements in version 2:

Section 5.1 "The practical measurement of the redshift in the spectrum" has been completely revised and supplemented, among other things by a synthetic quasar spectrum with several redshifted wavelength scales from z=1 to z=4.

1 Preface

Just within the last 100 years cosmology has emancipated itself from a forum of speculations and confessions and developed into a recognized, exact science [5]. The first phase from about 1920 to 1950 was still dominated by prestigious debates. Already in the late 1920s it was decided that Messier's galaxy world, including Andromeda M31, does not belong to the Milky Way but forming independent galaxies. However, the controversy lasted until the 1960s, whether the universe is static or dynamically expanding, triggered by a "Big Bang"!

Just 6 years before the first manned moon landing, in 1963 the redshift of the quasar 3C273 was discovered, which at that time was considered as huge, causing at first some confusion and headache. Later it became clear that the order of magnitude for the observable distances had now suddenly expanded from "hundreds of millions-" to "billions of light years". Just one year later, two telecommunications engineers made the groundbreaking, accidental discovery of the cosmic microwave background radiation that has already been predicted since 1933 (see cover picture). This ultimately sealed the definitive end of the "Steady State Theory". Some decades later followed the spectacular recordings of gravitational lenses and from the highly successful Hubble Deep Field campaign, followed in 1998 by the discovery of the accelerated expansion and the development of the currently favored ΛCDM model.

Despite sophisticated theories and methods, as well as an impressive park of "high-tech" instruments, we are faced today with the unsolved question of the nature of "dark matter" and "dark energy". Whether something existed before the "Big Bang" and what ultimately triggered it will probably for a long time remain a philosophical question.

Thanks to impressive progresses in digital photography, affordable spectrographs, and the increasingly possible access to large aperture telescopes, even amateur astronomers are now able to measure "cosmologically relevant" distances. These include distances of at least several 100 million light years, where the expansion of space already clearly dominates over the Doppler effects, caused by the proper motion of galaxies. Here follows a citation of Georges Lemaître from 1927, who was the very first, and long before Edwin Hubble, to recognize the decisive aspect of cosmological redshift [2]:

The redshift of galaxies is not due to the Doppler Effect, but by the expansion of space...

Also in this script, if nothing else is noted, the redshift is always to be understood as a result of the space expansion, and not as a result of Doppler effects. For this "cosmological order of magnitude", the distances to Messier's galaxy world with ≤80 million light years, are still too short, although the measured radial velocities already show a clear trend towards space expansion [27].

The "cosmological showpiece" for amateur astronomers is the already mentioned quasar 3C273 in the constellation Virgo [31]. With an apparent brightness of $m_V \approx 12.8$ and a z-value of 0.158 the object can be measured spectroscopically with telescopes from an aperture of ≧ 8 inch. Instruments from about ≧14 inches, even allow the measurement of some extremely bright quasars in the range up to z ~ 4! The top object in question is the quasar APM 08279+5255 in the constellation Lynx. With an apparent brightness of $m_V \approx 15.2$ and a z-value of 3.9 it is the brightest known object in the visible part of the universe so far, probably with a gravitational lensing effect as a "luminosity booster". The spectral atlas [28] contains a list of further such objects.

Among others the motivation for this script was to gain an overview to the "zoo" of cosmological distance concepts and particularly their practical application in astronomy. Further the cosmological parameters and distance measures which can be determined by the measured wavelength λ, and the rest wavelength λ_0 of an identified spectral line. The results are now summarized here and should help to better understand and interpret the own measurement results. Further, focused on practical astrospectroscopy, the book "Spectroscopy for Amateur Astronomers..." [27] outlines some cosmological interdependencies, which are now supplemented here.

The mathematical bases of the cosmological models are complex. Moreover, for the 3D-trimmed human brain the space time relationships are accessible just with simplified thought models. The author hopes that the necessary compromises have never exceeded the tolerable. For physically "watertight" derivations and definitions, please refer to the professional publications as well as the lecture notes of Laura Baudis [5] [6] and the presentations of Max Camenzind [1] [2] [3] [4].

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2 The Cosmological ΛCDM Model

2.1 Overview

To this topic just a short overview is provided here, limited to what is necessary to understand the following sections. All parameters, distance measures and also the calculation tools introduced here are based on the currently favored standard model "ΛCDM". Here Λ (Lambda) means the "cosmological constant" and "CDM": Cold, Dark Matter. At present (2020), most cosmologists assume that the universe is not or just slightly curved and therefore "flat". Viewed over large scales, it fulfills the requirements of the cosmological principle, i.e. it can be considered as homogeneous and isotropic (without a preferred direction) [5].

The so-called *Friedmann-Lemaître-Robertson-Walker Metric* (FLRW) is a solution of the field equations of Einstein's General Theory of Relativity GTR [9]. Based on this, the ΛCDM model [2] was developed from about 1998, among others by including Λ, the still enigmatic and apparently "anti gravitational" acting "dark vacuum energy".

2.2 Model Parameters and Variables

With just a few parameters, the ΛCDM model describes the expanding universe from the "Big Bang" to the future and shows a good correlation with current measurements in various wavelength ranges. The available calculation tools (section 5.2) mainly use as variables:

- the Matter density parameter Ω_m
- the Vacuum energy density parameter or the "Cosmological constant" Ω_Λ from Einstein's Genral Theory of Relativity GTR, which today mainly represents the effects of "dark energy"
- the Hubble constant $\mathrm{H}(0)$.

2.3 The Cosmological Fundamental Plane in the ΛCDM Model

Together with the model parameters, these variables determine not only the shape but also the expansion rate and age of the universe. With the additional Curvature parameter Ω_K the following simple equation results, which is also called the "Cosmologic Fundamental Plane" [4].

$$\Omega_\mathrm{K} + \Omega_\mathrm{m} + \Omega_\Lambda = 1 \qquad \{1\}$$

For the "flat" universe of the ΛCDM model therefore applies:

$$\Omega_\mathrm{K} = 1 - \Omega_\mathrm{m} - \Omega_\Lambda \approx 0.$$

This condition is met for the ΛCDM model by the following currently accepted parameters:

$$\Omega_\mathrm{m} \approx 0.27,\ \Omega_\Lambda \approx 0.73 \quad \rightarrow \quad \Omega_\mathrm{m} + \Omega_\Lambda \approx 1.$$

2.4 Hubble Parameter $\mathrm{H(t)}$ and Hubble Constant $\mathrm{H(0)}$

The Hubble parameter H(t) is the relative measure for the expansion velocity of the space. For example, $\mathrm{H(t)} = 74\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ means that at the time t a distance of 1 Mpc is increasing every second by 74 km. This parameter is variable over time and also forms the proportionality factor between the redshift and the distance of the galaxies in our nearer vicinity of the universe (section 3.7). The so-called Hubble constant $\mathrm{H}(0)$ shows as a special case, i.e. limited to our local area of the universe, the current value of the Hubble parameter and plays a decisive role in cosmological models. For still enigmatic reasons, two values for $\mathrm{H}(0)$ are currently crystallizing with a difference of about 10%, both with an uncertainty of just <2% percent [30].

The first value is based on the relatively close vicinity, i.e. in the Magellanic Clouds, on current, mainly photometric measurements of Cepheids and in distant galaxies on the "standard candles" of the Type Ia supernova. This campaign yielded about $74\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ (Adam Riess et al.). Already in the 1920s Hubble applied the pulsation-variable Cepheids to determine the distance of M31.

The second value of about $68\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ is based on the latest analyses of the cosmic microwave background, with data from the European Planck satellite.

Currently, theorists are working intensively on the explanation of this discrepancy [30] - with possible implications for the currently established standard models and the sure outlook for Nobel prizes! The cosmological calculation tools (section 5.2) as default value mostly apply $\mathrm{H}(0) \approx 68\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

2.5 The Cosmological Time t

The cosmological time t forms the time axis and the measure for the course of the space expansion. It starts at the hypothetical "Big Bang" with $t = 0$ and is measured by the clocks of fictional "Comoving observers", remaining without motion, i.e. resting in the expanding universe or embedded in the "Hubble Flow". The classical explanatory model here is the surface of a balloon with painted dots diverging when inflated. The cosmological time of the present is designated with $\mathrm{t}(0)$. It amounts to about 13.7 Gyr (Giga years) and corresponds to the so-called "Age of the Universe".

Note: In the expanding universe, at a certain cosmological time t - and regardless of their location - all the "comoving observers" measure the same time. The effect of time dilatation, which is effective in the Special Theory of Relativity STR, due to observers moving in different directions, is irrelevant here.

2.6 The Hubble Time t_H

The Hubble time t_H corresponds to the reciprocal value of the Hubble constant and may serve as a rough approximation for the age of the universe [27].

$$t_H = \frac{1}{H(0)} \qquad \{2\}$$

If space expansion in an empty universe would be constant, t_H would be equal to the current Age of the Universe $\mathrm{t}(0)$ of 13.7 Gyr, i.e. the time elapsed since the hypothetical "Big Bang" (section 2.5). At present, however, in addition to the "baryonic" matter, which is familiar in everyday life, now dark matter and dark energy are postulated, influencing the expansion of space. This is why the age of the universe differs from the Hubble time (determination see [27]). Based on $\mathrm{H}(0) = 74\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ the Hubble time yields $\mathrm{t_H} \approx 13{,}3\ \mathrm{Gyr}$.

2.7 The Cosmological Scale Factor a

The scale factor a, stands for the relative size of the universe and describes the stretching effect of space expansion as a function $a = f(t)$, increasing monotonically along the time axis. Hypothetically starting with the "Big Bang" with $a = 0$, the convention or normalization applies to the present $\mathrm{t}(0)$:

$$a(t0) = 1 \qquad \{3\}$$

In conclusion, this means:

$a = 0$	"Big Bang"
$a < 1$	Past
$a = 1$	Present
$a > 1$	Future

The expansion of space increases the physical distances between the objects embedded in the so-called "Hubble Flow". This can be illustrated by the already mentioned model of the balloon surface, which increases when inflated.

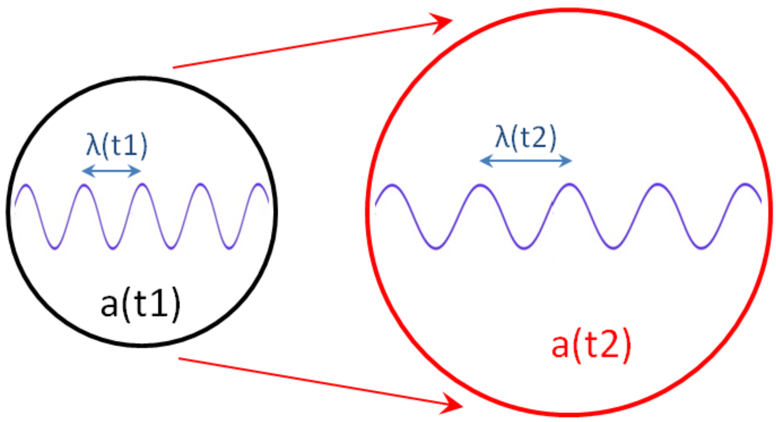
For the cosmologically induced stretching of a certain distance, measured at two different points in time t1 and t2, the simple, proportional relationship applies:

$$\frac{D(t1)}{D(t2)} = \frac{a(t1)}{a(t2)} \qquad \{4\}$$

$\mathrm{D(t1)}$:	Distance at time t1
$\mathrm{D(t2)}$:	Distance at time t2
$\mathrm{a(t1)}$:	Scale factor at time t1
$\mathrm{a(t2)}$:	Scale factor at time t2

2.8 Scale Factor and Redshift z

According to Einstein's ART the wavelength λ of light expands proportionally to the expansion of space {4}.



Thus, by means of spectroscopy the stretching of the wavelength becomes immediately measurable and a cosmological scale factor a for the time t can be calculated very easily with spectroscopically obtained data of the present:

1. Directly based on the wavelengths of an identified spectral line:

$$\alpha(t) = \frac{\lambda_0}{\lambda} \qquad \{5\}$$

λ_0: Rest wavelength of the line, i.e. originally emitted by the object at time t
λ: Currently measured wavelength of the line

2. Based on the calculated z-value (section 3.3):

$$\alpha(t) = \frac{1}{1+z} \qquad \{6\}$$

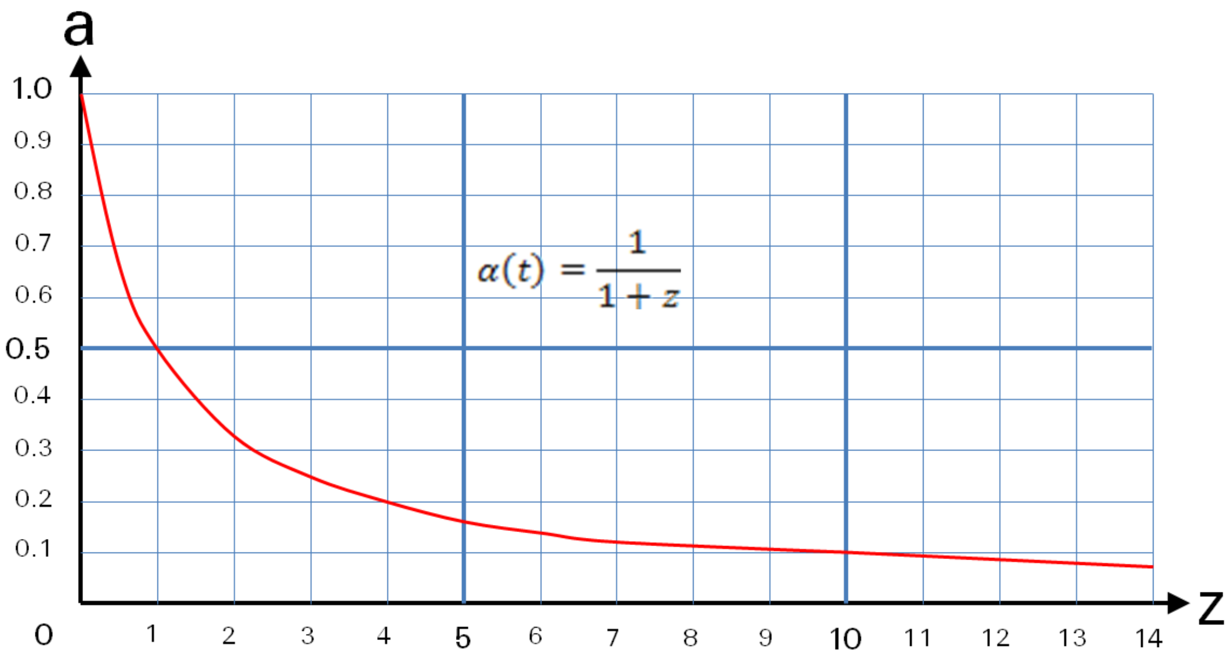
Probably because of their simplicity, these equations are not covered by all cosmological calculation tools (section 5.2), so in such cases the pocket calculator must be applied.

2.9 Slow Motion Effect due to Expansion of Space

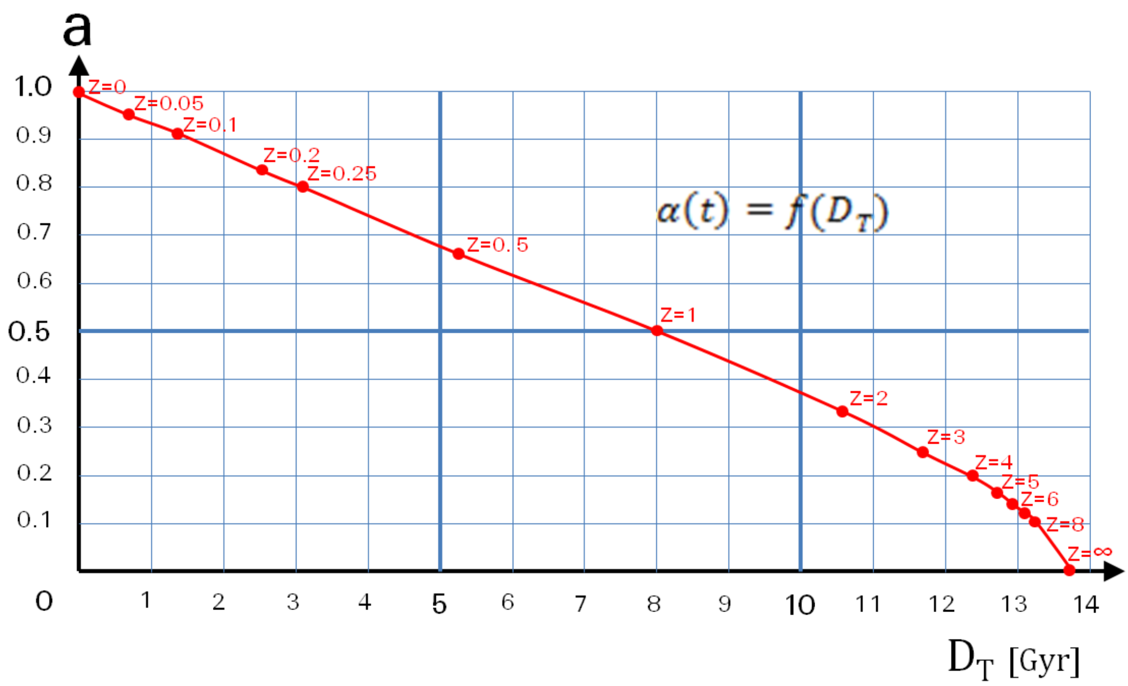
The expansion of the space, and thus also of the wavelength, generates a slow-motion effect. Thus, the brightness curve of a supernova, observed in a "High-z" quasar, runs much slower. This also disproofes the "tired light theory", proposed in 1929 by Fritz Zwicky. Thus, the redshift cannot be explained by a loss of photon energy.

2.10 The Development of the Scale Factor a over the Time

If applying the z-value as a time axis, the function $\alpha(t) = f(z)$ can immediately be plotted according to equation {6}, without applying any cosmological model. However, the course of this graph is just of limited expressive power because the z-value is not a linear measure of time.



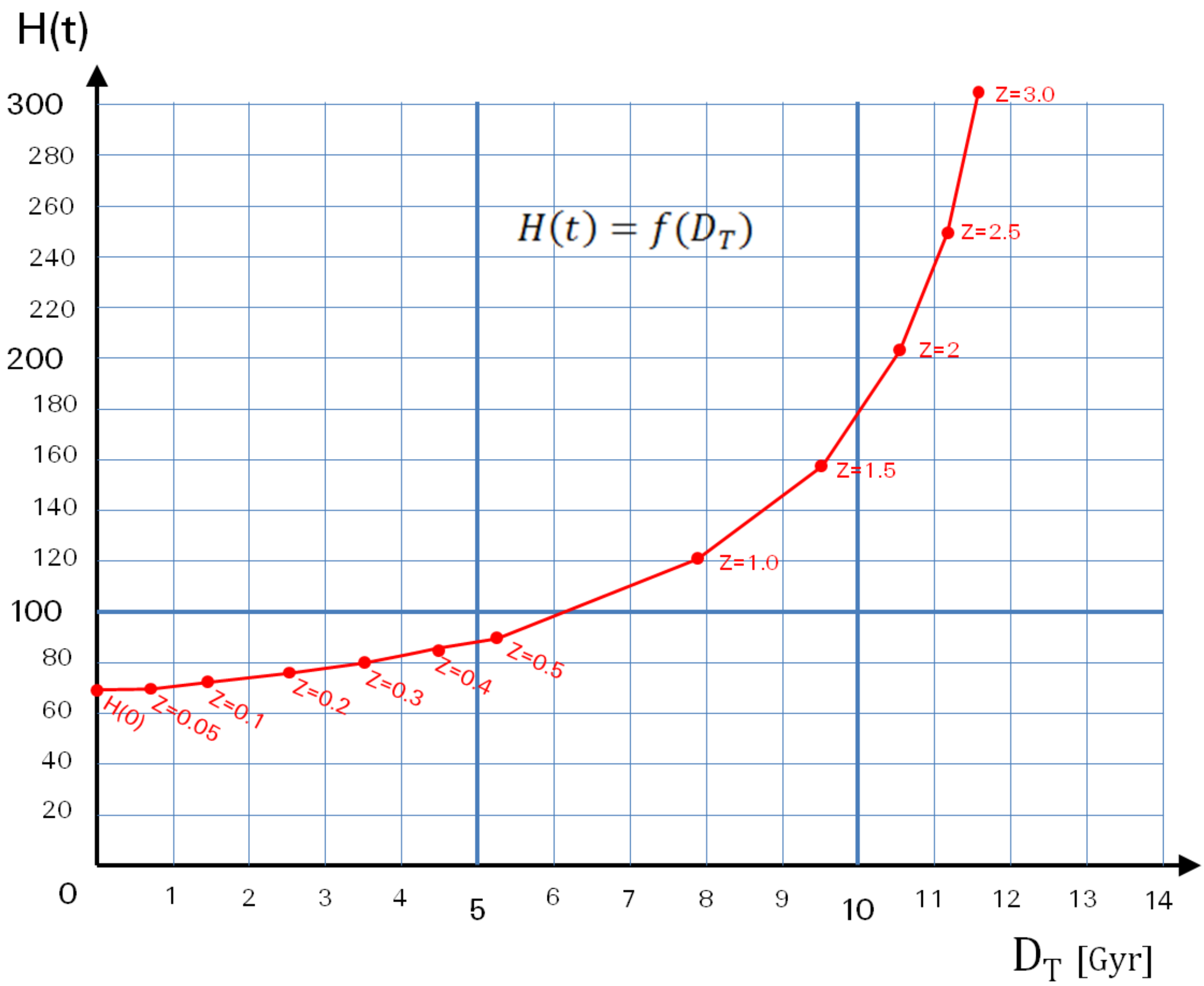
This deficiency can be eliminated by using the linear running Light Travel- or lookback time D_T (section 3.6) as a time axis and displaying the development of the scale factor as a function $\alpha(t) = f(D_T)$. For this purpose, the z-values (red) must first be converted into Light Travel Time D_T, applying a cosmological calculation tool [24]. D_T can be interpreted both, as a distance- or a time axis t. Shown in this way, this function runs more or less linearly between 0 and about 10 Gyr ($z \approx 2$) followed by a significantly steeper drop towards the "Big Bang". This section of the curve shows, that the expansion of space was obviously much greater in the early days of the Universe. According to the ΛCDM model, the scale factor will increase again in the future as a result of the accelerated expansion (section 2.12).



2.11 The Development of the Hubble Parameter over the Time

The following diagram shows that the Hubble parameter has been decreasing since the early days of the universe and currently seems to fall towards a fixed size > 0, what is actualy attributed to the "dark energy". However, as a result of the "accelerated expansion", the Hubble parameter, like the scale factor, should even increase again in the future (section 2.12).

Also, for this diagram, the z-values (red) were first converted into Light Travel Time D_T applying a cosmological calculation tool [24]. The corresponding values for the Hubble parameter $[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$ have been calculated by the tool of Nick Gnedin [21].



2.12 The Cosmological Definition of the Hubble Parameter

For mathematically interested follows here the derivation of the cosmologically important relationship between the Hubble parameter $\mathrm{H(t)}$ and the scale factor $\mathrm{a(t)}$, i.e. the function $H(t) = f(a)$.

Section 2.9 has already shown how the scale factor changes over time, i.e. $a(t) = f(t)$. This defines now also the directly related Hubble parameter $\mathrm{H(t)}$. In line with the Hubble-Lemaître law (section 3.7), $\mathrm{H(t)}$ acts here, instead of $\mathrm{H(0)}$, as a generally valid proportionality factor between the expansion velocity v_r and the distance D.

$$v_r = H(t) \cdot D$$

The expansion velocity of the space v_r can also be expressed as a change of a distance per time unit with the differential quotient $\mathrm{dD/dt}$.

$$\frac{dD}{dt} = H(t) \cdot D \qquad \rightarrow \qquad H(t) = \frac{\frac{dD}{dt}}{D}$$

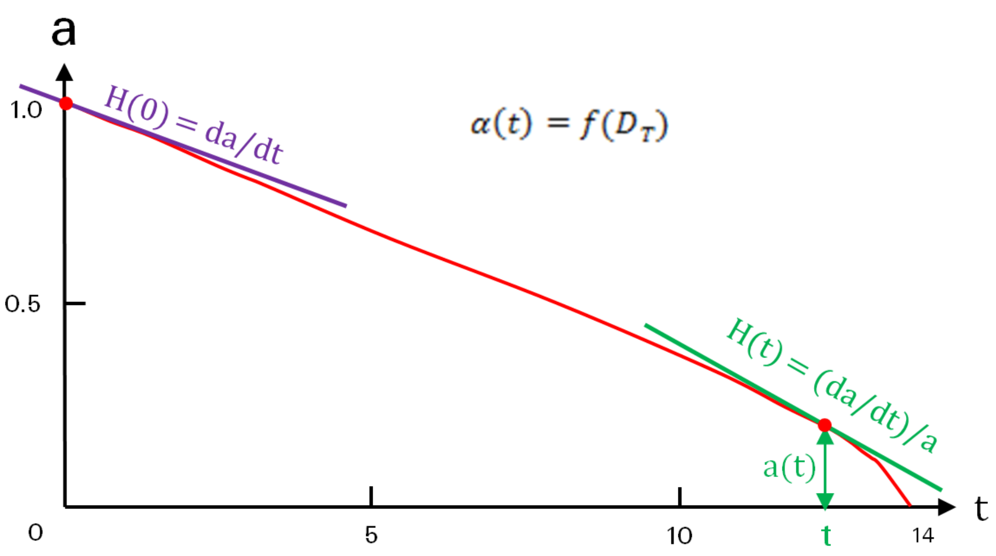
In physics differential quotients containing time derivatives, are mostly abbreviated with a superscript point above the differential:

$$\frac{dD}{dt} = \dot{D} \qquad \rightarrow \qquad H(t) = \frac{\dot{D}}{D}$$

According to equation {4} distances and scale factors behave proportionally. Thus, if the distance D is replaced by the scale factor a, this results in the famous differential equation, which cosmologically defines the Hubble parameter, and finally also the so-called "Hubble Flow":

$$H(t) = \frac{\dot{a}}{a} \qquad \{7\}$$

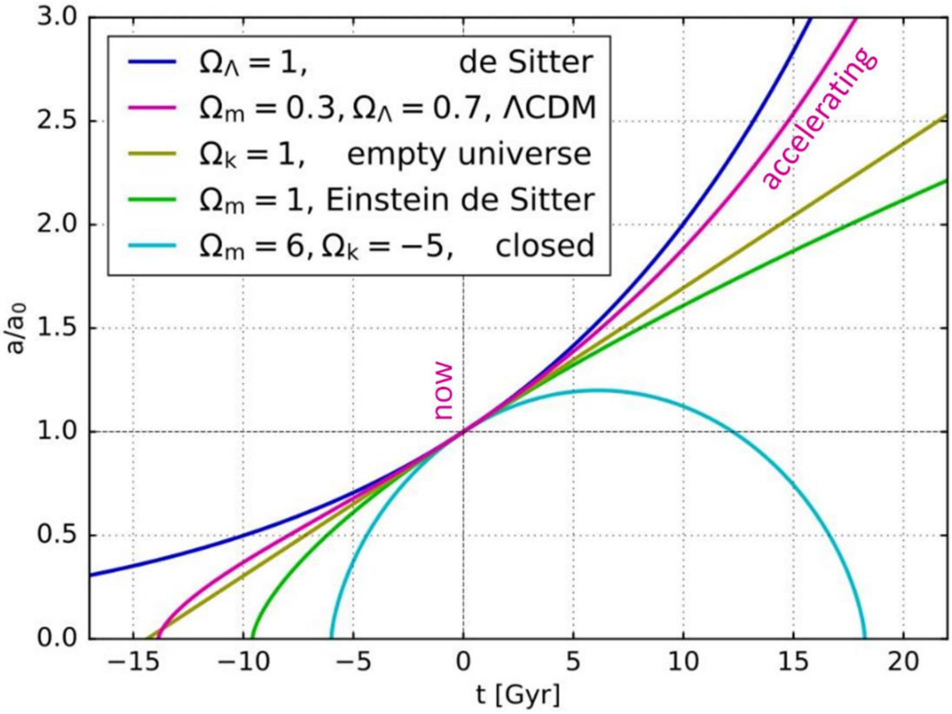
Thus, the Hubble parameter is equal to the time derivative of the scale factor a, and normalized to the corresponding scale factor $\mathrm{a(t)}$. With the diagram $\alpha(t) = f(D_T)$, simplified from section 2.9, this relationship can be demonstrated graphically. The differential quotient $\dot{a} = \frac{da}{dt}$ corresponds to the slope of the green tangent at the red function graph. Finally, this value must still be divided by the scale factor $\mathrm{a(t)}$. The violet tangent shows as a special case, the Hubble parameter of the present $\mathrm{t} = 0$, where the scale factor is $\mathrm{a} = 1$ and the Hubble parameter $\mathrm{H(t)}$ becomes the Hubble constant $H(0) = \dot{a}$.



2.13 The Scale Factor in Cross Comparison of Cosmological Models

The following, somewhat supplemented diagram from Wikimedia Commons [15], shows for different parameterizations of the Friedmann equations, the time development of the scale factor a, from the past up to the future. Some of the historical precursor models bear really sounding names!

The purple graph displays the course of the currently favored ΛCDM model, already shown in section 2.9. However, the time axis points here in the opposite direction, with negative values for the past and positive values for the future. It is impressive to see how the "ΛCDM graph" for the scale factor shows towards the future an <u>accelerated expansion</u> and how the present ("now") seems to be near a turning point.



3 Distance Measures in Cosmology

3.1 Distances in our Everyday Life

Already in the Euclidean 3D space of our everyday life there are several possible measures for very long distances, which depend e.g. on the curvature of the earth and thus also on the space. For example, the shortest distance between two cities in Europe and Australia is measured along a great circle (orthodrome) on the surface of the globe. This can also be demonstrated with a thread, stretched on a globe between these places. This measurement is of practical importance in everyday life and is needed, among other factors, for a rough estimate of the duration and fuel consumption of a long-haul flight. However, the geometrically shortest possible connection would run along the chord of this great circle segment, and in this case even through the earth's core.

3.2 Distances in the Large-Scale Universe

The conditions in the large-scale universe are much more complex. Here, the propagation of light is influenced, among other things, by the expanding geometry, determined by space-time. The distance therefore also depends on the time of measurement. Therefore, no "trivial" distance measure exists here, so the application of complex cosmological models is required (section 2). The corresponding distance units are, besides the z-value, mostly the light year [ly], Mega light year [Mly], Giga light year [Gly] or the parsec [pc], where 1pc ≈ 3.26 ly. In astrophysics, in the cosmologically relevant range, several distance measures are distinguished. Their selection is determined by the respective application.

3.3 The z-value – The Universal Distance Measure of the Redshift

The so-called z-value of the redshift is the only quantity within the cosmologically relevant framework that can be measured absolutely and thus remains independent of any corresponding models. It can be determined, even by amateurs, very easily and with high precision, from the shift of a spectral line within a wavelength calibrated spectrum [27].

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} \qquad \{8\}$$

λ: measured wavelength of an identified spectral line
λ_0: Rest wavelength of the spectral line (originally emitted by the object this way)

The z-value thus enables an absolute comparison of distances between different objects, but with the disadvantage to be neither proportional to distances nor to time periods (section 2.8). As a result of the finite speed of light, this measure therefore extends over the space- and time dimension and is therefore in addition a measure for the past. The extreme values of z are of cosmological importance:

$z = 0$: →	$\mathrm{t} = \mathrm{t}(0) \approx 13.7\ \mathrm{Gyr}$	The Present
$z = \infty$: →	$\mathrm{t} = 0$	The "Big Bang"
$z \approx 1089$: →	$\mathrm{t} \approx 380'000\ \mathrm{yrs}$	The Microwave background radiation

The microwave background radiation at $\mathrm{z} = 1089 \pm 0.1$ [14] forms an opaque barrier at least for optical observations.

Application: Within the cosmologically relevant distance range these properties make the dimensionless z-value the most applied distance measure in scientific publications. As a measured value, it has also a key function in determining the subsequently introduced distance measures. However, this is only possible with the help of cosmological models and corresponding tools.

3.4 The Proper Distance D_P - The "Physical" Spacing between Objects

The Proper Distance D_P can be thought as the length of a thread that is stretched between two objects at exactly the same cosmologic time t. Since this measure does not remain constant, but stretches over time in proportion to the expansion of space, in this thought model the thread would immediately break. At the cosmological time of the present $t = t(0)$ D_P corresponds to the current, "real" spacing between objects. However, due to the finite speed of light and the identical time t(0), the measured object is not observable at the present location. Therefore, the Proper Distance just extends over the space- but never over the time dimension.

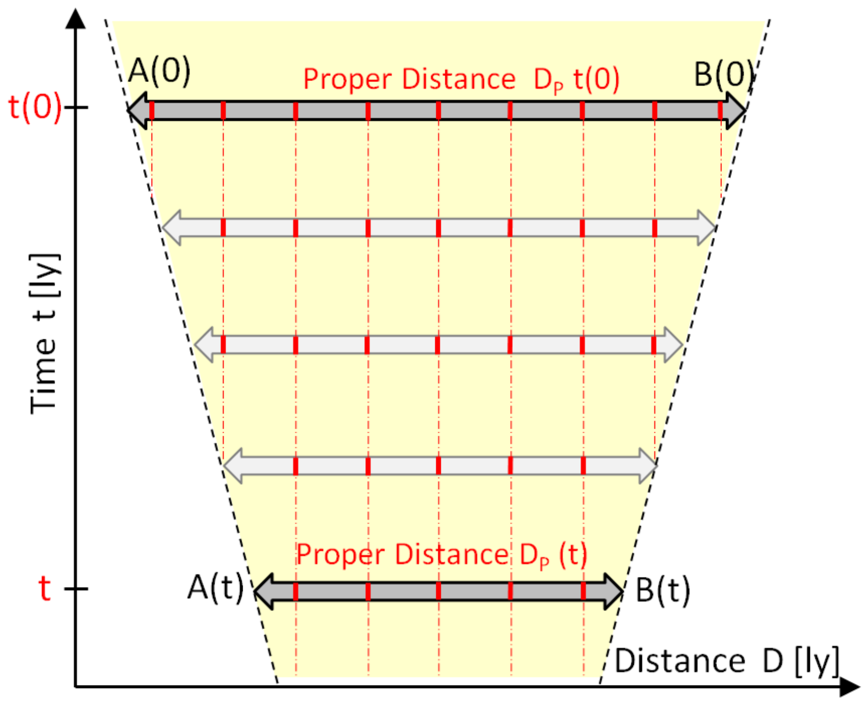
Application: This measure can just be determined indirectly by the redshift z and with cosmological models. It corresponds most closely to our conventional concept of distance. For this very reason D_P would be the right choice for popular scientific articles [8], which however prefer the massively shorter distance of the Light Travel Time D_T (section 3.6). The current Proper Distance to the "Big Bang", i.e. to the observation or particle horizon (section 4.1), measures approximately 46.6 Gly. Because the expansion of space is here taken into account, in this extreme case D_P equals almost 3.5 times the corresponding Light Travel Time D_T of 13.7 Gyr (section 3.6).

According to equation {4}, the distance $D_P(t0)$ of the present compared to the same distance in the past or future $D_P(t)$ behaves proportional to the corresponding cosmological scale factors $a(t0)$ and $a(t)$. In addition to the present applies the convention $a(t0) = 1$.

$$\frac{D_P(t0)}{D_P(t)} = \frac{a(t0)}{a(t)} = \frac{1}{a(t)} \rightarrow$$

$$D_P(t) = D_P(t0) \cdot a(t) \qquad \{9\}$$

As a graphical compromise, the following space-time diagram with a horizontal distance and a vertical time axis demonstrates the development of the Proper Distance D_P over time. The horizontal grey arrows on the yellow colored area schematically show the increase of the past Proper Distance A(t) - B(t) to the present value A(0) - B(0). The scale remains constant over time – note the distances between the red scale marks in the diagram.



3.5 The Comoving Distance D_C – With Neutralized Space Expansion

This fairly abstract concept is based on a coordinate system, stretching proportional to the expansion of space. Thus, to the same extent, the scale is stretching and the distance measured between comoving objects remains constant over time. The Comoving Distance D_C gets absolutely measured at a certain point in time t, which is defined hereafter as the "present t(0)". Exclusively at t(0), D_C corresponds now exactly to the real Proper Distance D_P, according to equation {11}. If the space in the future is further expanding – or with a view to the past is shrinking, the measured value of the distance, determined at t(0), remains constant (see the red scale marks in the diagram). Thus the expansion of space is now practically "factored out" or "neutralized". Only the proper motion of objects, deviating from the theoretical rest position within the expanding "Hubble Flow", e.g. galaxies in a large cluster, can change D_C by small amounts over time.

Application: These properties are applied for special applications, e.g. for motion studies within large galaxy clusters by neutralizing the expansion of space. Also the comoving distance D_C just extends over the space- but never over the time dimension.

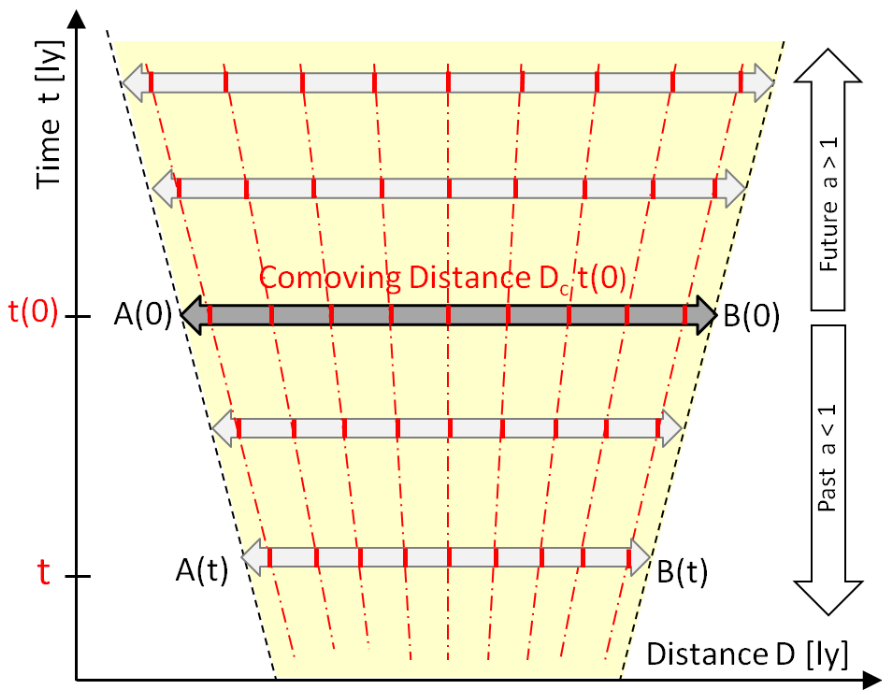
Caution: Deviating from this definition, in the professional literature almost everything that stretches or moves with the expansion of space, is sometimes referred to as "comoving". Thus, in addition to "Comoving time" and "Comoving volume", the time-variable Proper Distance D_P is also referred to as "Comoving Distance D_{Com}". This is also applied this way by the cosmological calculation tools. However, D_C corresponds in fact to the Proper Distance D_P at the respective measurement time. Hereafter in this context, the term "Proper Distance" will be consistently applied.

At a certain time t, the ratio of the Proper Distance D_P, to the Comoving Distance D_C, corresponds always to the cosmological scale factor $a(t)$.

$$a(t) = \frac{D_P(t)}{D_C} \quad \rightarrow \quad D_P(t) = a(t) \cdot D_C \qquad \{10\}$$

For the time of measurement, here in the presence $t = t(0)$ and with the corresponding scale factor $a = 1$, the following applies:

$$D_P(t0) = D_C(t0) \qquad \{11\}$$



3.6 The Light Travel Time D_T – The Distance into the Past

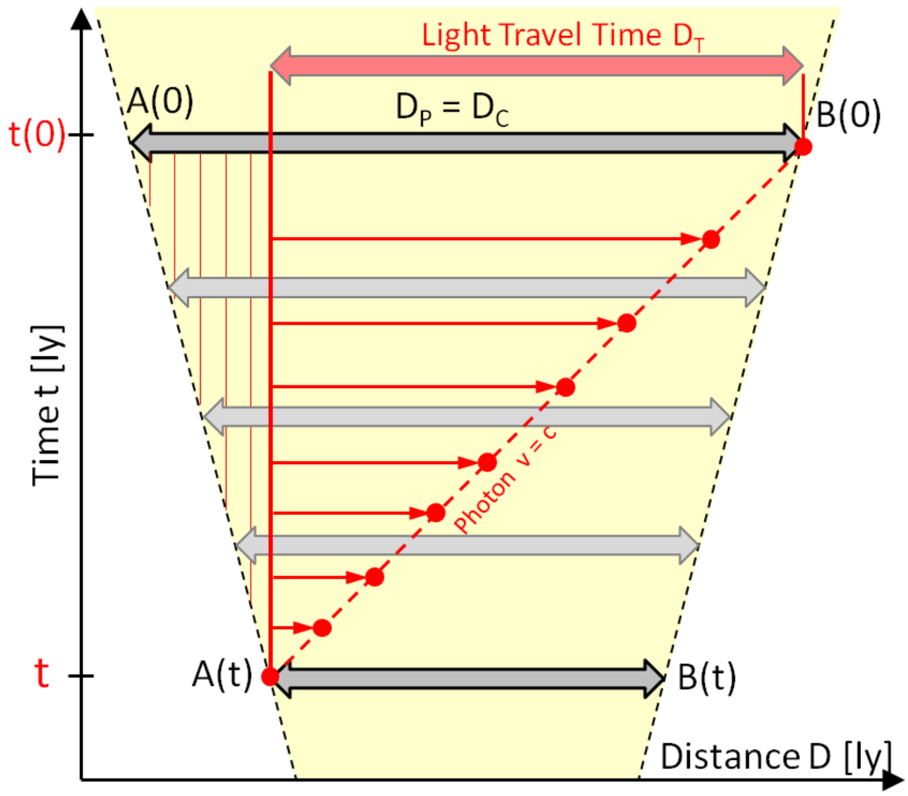
The Light Travel Time D_T or Lookback Time is the time required by the light of an object for the distance between the emission at time t and the present observation time $t(0)$.

$$D_T = t(0) - t \qquad \{12\}$$

In literature, the Light Travel Time D_T is always expressed as a time difference and mostly in [Gyr] (Giga Year). For extremely high z-values D_T approaches asymptotically the age of 13.7 Gyr. Multiplying the Light Travel Time D_T by the vacuum speed of light c gives the Light Travel Distance [Gly] (Giga Light year) [2].

$$Light\ Travel\ Distance = c \cdot D_T \qquad \{13\}$$

As a graphical compromise, the following (Minkowski) space-time diagram shows the ratio of the Light Travel Time D_T to the already introduced measures D_P and D_C [8]. The space expanding over time is also here symbolized by the horizontal, grey arrows on the yellow-colored area. The past Proper Distance A(t) – B(t) is growing to the present value A(0) – B(0).



To illustrate the Light Travel Time D_T, at the time t a red photon is sent from the position A(t) in the direction of B(t). In the vertical time dimension t the horizontal red arrows show how the photon moves at the speed of light through the space, expanding here with v < c and becomes finally detected at the time t(0) at position B(0). It is important to note that the space also expands in the opposite direction to the movement of the photon, which corresponds to the red shaded triangular area. Due to this effect, the Light Travel Time D_T is stretched much less by the space expansion and therefore always shorter than D_P or D_C.

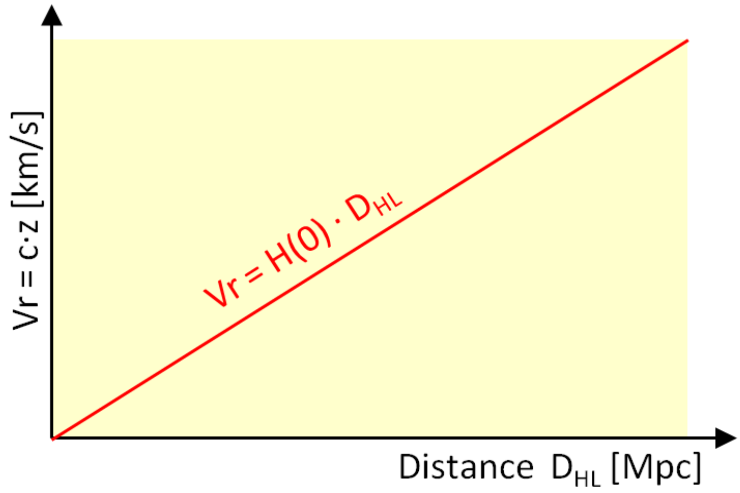
<u>Application:</u> This measure can just be determined indirectly by the redshift z and with cosmological models. While the z-value dominates in scientific publications, the Light travel time is the most commonly used distance measure in popular scientific articles. In contrast to the Proper- or Comoving Distance, this measure extends over the space- <u>and</u> the time dimension.

3.7 The Distance D_{HL} in the Hubble-Lemaître Law

The historically important Hubble-Lemaître law [10] [29], postulates an empirically determined, linear relationship between the distance of the galaxies D_{HL} and the spectroscopically determined, apparent "escape velocity" v_r. The Hubble constant $H(0)$ forms the proportionality factor (section 2.4). Georges Lemaître and Edwin Hubble discovered this relationship at the end of the 1920s and independently of each other. However, the values for $H(0)$ of >500 km s^{-1} Mpc^{-1} determined at that time, were still about one order of magnitude too high!

$$v_r = H(0) \cdot D_{HL} \qquad \{14\}$$

v_r: Radial or apparent "escape velocity" [km s^{-1}]
D_{HL}: Distance to object [Mpc]
$H(0)$: Hubble constant [km s^{-1} Mpc^{-1}] (section. 2.4)



To determine the Hubble constant, D_{HL} is (and was historically) determined photometrically with so-called "cosmic standard candles" (section 2.4). The radial velocity v_r can spectroscopically be determined by the z-value (c = speed of light).

$$v_r = c \cdot z \qquad \{15\}$$

In the cosmologically close vicinity the distance D_{HL} can approximately be regarded as the Proper Distance D_P, since the Luminosity Distance D_L (section 3.8), used for the calibration of $H(0)$, has approximately the same length (section 3.10).

Application: This measure can just be determined indirectly by the redshift z and with the Hubble constant $H(0)$. The proportionality of the Hubble-Lemaître law applies only to the cosmological close vicinity of some 100 Mly, which limits the scope of application for distance determination accordingly. For Messier's galaxy world, with distances of ≤80 Mly, this is still the case. However, in this "close range", the Doppler Effect, due to the proper motion of the galaxies, significantly overprints here the still small cosmological expansion of space [27]! A famous example is the Andromeda galaxy M31, approaching the Milky Way with a blue-shifted spectrum, i.e. about 300 km/s. With increasing z-values, the distances are massively overestimated by D_{HL} (section 3.11).

naive Hubble: For comparison purposes, the linear extrapolation of the Hubble-Lemaître law up to high z-ranges can make sense and is often referred to in this context as "naive Hubble" (section 3.10).

3.8 The Luminosity Distance D_L – The Photometric Measure

The Luminosity Distance D_L is a "photometric measure" and is based on the known or estimated absolute brightness M_V of an observed object. The distance can be determined directly by comparison with the measured, i.e. apparent brightness m_V. Among other effects, this is based on the "dilution" of the photon flux due to the spherical light propagation. The difference between apparent and absolute brightness, expressed in magnitudes [mag], is called the Distance Module μ [27] and is provided as a result by some of the cosmological calculation tools.

$$\mu = m_v - M_v \quad [mag] \quad \{16\}$$

The distance D_L can be calculated as follows [27]:

$$D_L = 10^{0.2(m_v - M_v)+5} \quad [pc] \quad \{17\}$$

As a result of space-time effects and the extinction due to dust etc., the distances according to the simple equations {16} and {17} are massively overestimated, which requires corresponding corrections. In a cross-comparison (section 11), luminosity distances are therefore always the longest.

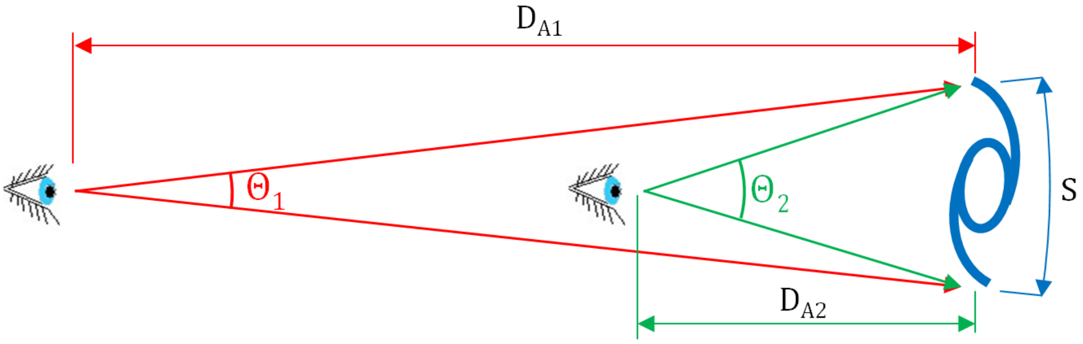
The contribution of spectroscopy to the direct determination of the Luminosity Distance is limited to the estimation of the absolute brightness M_V of the object, based on the spectral signatures. For stellar objects usually the spectral and luminosity class is determined. For spiral galaxies, the rotation speed is determined, which is coupled with the absolute brightness by the empirical Tully-Fisher relation. For this purpose, the broadening or even splitting of certain spectral lines is evaluated. Such a spectral signature can be seen in the Spectral Atlas [28] in the profile of the LINER galaxy M94 (Plate 55). For elliptical galaxies, this is supplemented by the Faber-Jackson relationship. In the field of radio astronomy, the emission of neutral hydrogen at the wavelength of ~21cm is measured.

Application: This measure can be determined photometrically or indirectly by the redshift z and with cosmological models. D_L is generally applied to measure distances in the near vicinity but also in the cosmologically relevant range. D_L was applied historically, but also still today, as an important element for the determination of the Hubble Constant. In contrast to the Proper- or Comoving Distance, this measure extends over the space- and the time dimension.

3.9 The Angular Diameter Distance D_A – The Measure of the Past

1. The direct measurement of the angular diameter distance

Spectroscopy plays hardly any role in the direct determination of the Angular Diameter Distance D_A. Here it is based on the known or estimated absolute diameter S of the observed object. The distance D_A, e.g. to a galaxy, can be determined applying simple angular laws by comparison with the optically measured apparent angular diameter Θ of the object.



$$D_A = \frac{S}{\Theta} \qquad \{18\}$$

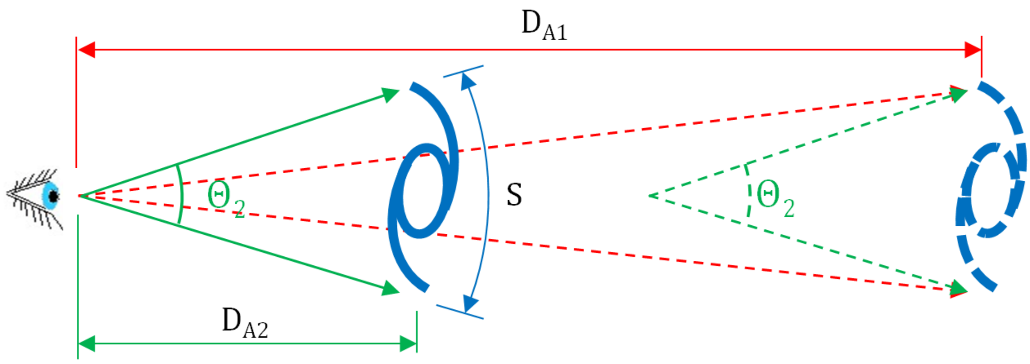
S: Real diameter of the object [ly, pc]
Θ: Measured angular diameter in radians [rad]
D_A: Distance to the object [ly, pc]

Conversion from degrees [°] to radians [rad]:

$$Radian\ [rad] = \frac{Degree\ [°] \cdot \pi}{180}$$

2. D_A – The measure of the past

Due to our unavoidable look into the past, the angle Θ, measured in the sky, does not correspond to the present, but to the past time t of light emission, where the universe was even smaller and the distances (D_{A2}) were much shorter. At today's larger distance D_{A1} we therefore measure the object not at Θ_1, but, although much fainter, with the formerly larger, green angle Θ_2.



With this method we therefore do not measure the present distance D_{A1} but the past and thus shorter distance D_{A2}, corresponding to the former Proper Distance $D_P(t)$:

$$D_{A2} = D_P(t)$$

D_{A2} can therefore also be calculated with equation {9} from the present Proper Distance $D_P(t0)$ and the scale factor $a(t)$:

$$D_{A2} = D_P(t0) \cdot a(t) = \frac{D_P(t0)}{1+z} \qquad \{19\}$$

In cosmology, this term is therefore also called "Angular Distance".

3. Consequences

- Paradoxically, by this effect, with increasing distance, the angular diameter of an object appears larger and larger and the expanding universe acts as a kind of "magnifying glass" [5]. At extremely high z values, i.e. close to the "Big Bang", the apparent size of an object would even approach the infinity. A perfect example of this is the cosmic background radiation at a redshift of z ≈ 1089 [14], reaching us today isotropically, i.e. from any direction.
- As a result of these effects, the angular diameter distance first grows with increasing distance. However, from z > 1.6, D_A becomes smaller again to even approach 0 at extremely high z-values (graphic section 3.11).
- In comparison, the angular diameter distance D_A is therefore still significantly shorter than the Light Travel Time D_T.

4. Application:

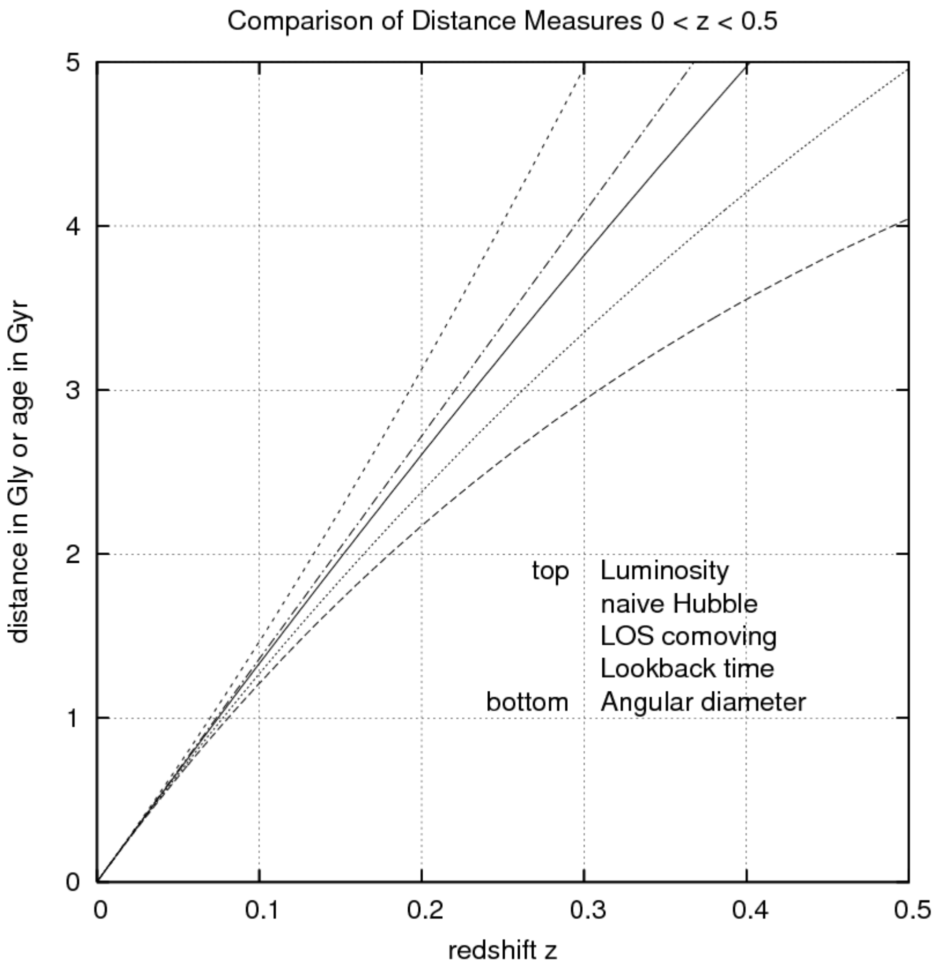
This measure can be determined directly by an optical angle measurement or indirectly by the redshift z and with cosmological models. Main application of D_A is the determination of distances in the past. It extends just across the space- but never to the time dimension.

3.10 Comparison of the Cosmological Distance Measures for 0 < z < 0.5

In our "closer cosmic vicinity" up to z ≈ 0.05, all measures presented here and by a comparable measuring accuracy, yield the nearly same distance. However, in the cosmologically relevant distance range, i.e. from several 100 Mly, differences become increasingly obvious. The following diagram (Wikimedia Commons – Distance Measures) shows this effect up to a distance of z = 0.5, corresponding to a light travel time of about 5 Gyr.

The legend in the diagram describes the graphs from top to bottom for all distances presented here:

Luminosity:	Luminosity Distance D_L
naive Hubble:	Hubble Lemaître D_{HL}, extrapolated up to z = 0.5
LOS Comoving:	Proper Distance D_P
Lookback time:	Light Travel Time D_T
Angular Diameter:	Angular Diameter Distance D_A



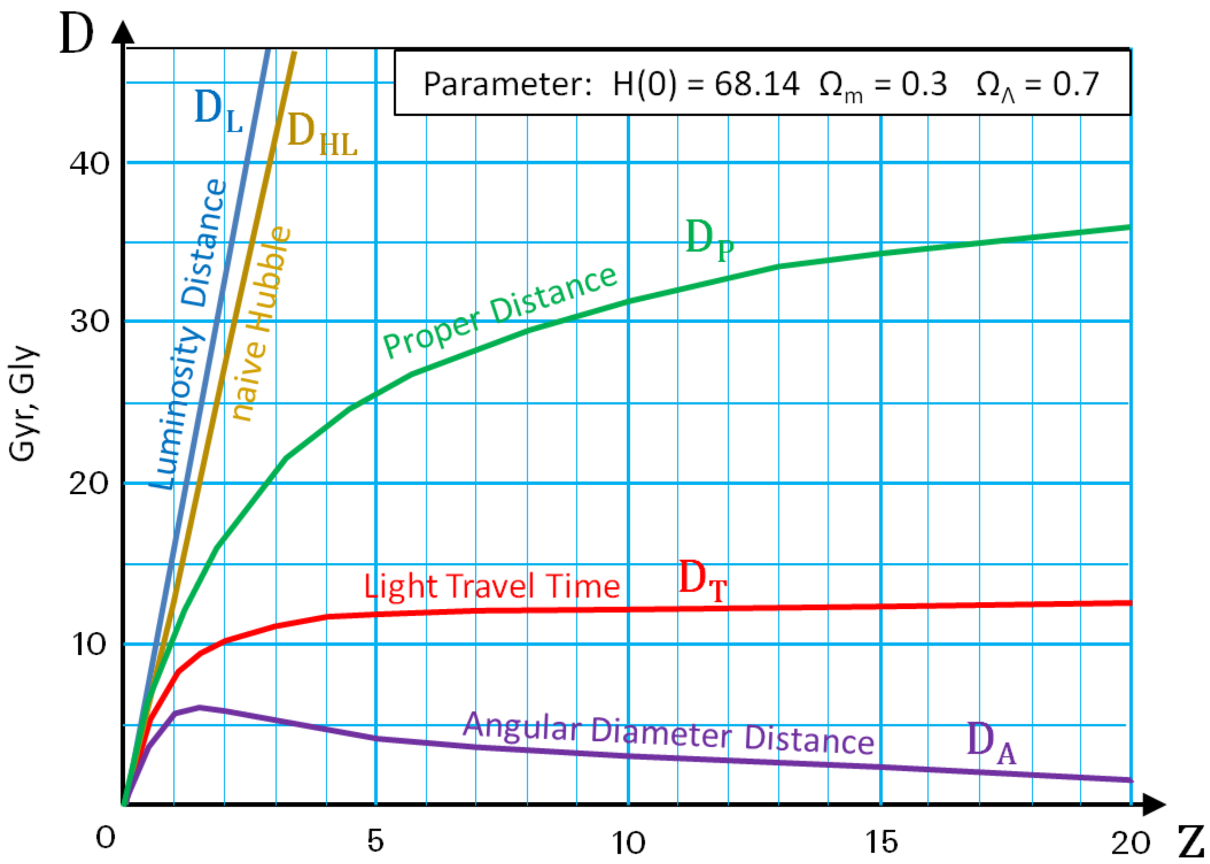
Comment:

Here it is well recognizable that:

- The Luminosity Distance D_L without appropriate corrections runs against an extreme and far too high value.
- The Proper Distance D_P is significantly longer than the Light Travel Time D_T.
- The Angular Diameter Distance D_A as a "Measure of the past" is clearly the shortest.

3.11 Comparison of the Cosmological Distance Measures for 0 < z < 20

The following diagram shows the same distance measures as in section 3.10, but here in a significantly extended time range up to z = 20.



This diagram shows among others that the Angular Diameter Distance D_A is the only one which is not a monotonically increasing function of the redshift. That means. at z ≈ 1.6 it reaches the maximum value and then for extreme z-values, it strives towards 0.

The distances, up to the present Observation- or Particle Horizon (section 4.1) i.e. at $z = \infty$ or the "Big Bang", amount:

- Proper Distance D_P: ~46.6 Gly
- Light Travel Time D_T: ~13.7 Gyr
- Angular Diameter Distance D_A: 0 ly

4 Cosmological Horizons

4.1 The Observation- or Particle Horizon

The Observation- or Particle Horizon is defined as a spherical surface. It forms the current limit to which the light has propagated which was emitted at the time of the Big Bang, and thus also from our former location. The current value amounts to 46.6 Gly as a Proper Distance or 13.7 Gyr as Light Travel Time. Due to the "dark energy" – so the current doctrine – it is constantly growing. Our Observation Horizon is the outermost limit where, from our current location, we can still observe objects whose light has been on its way to us since 13.7 Gyr. However, due to the impenetrable barrier of cosmic background radiation, this distance will for photons be shortened by about 380,000 yrs. At the same time t, every point in the universe is surrounded by its own Particle Horizon of exactly the same size, which may overlap with others but never can be identical.

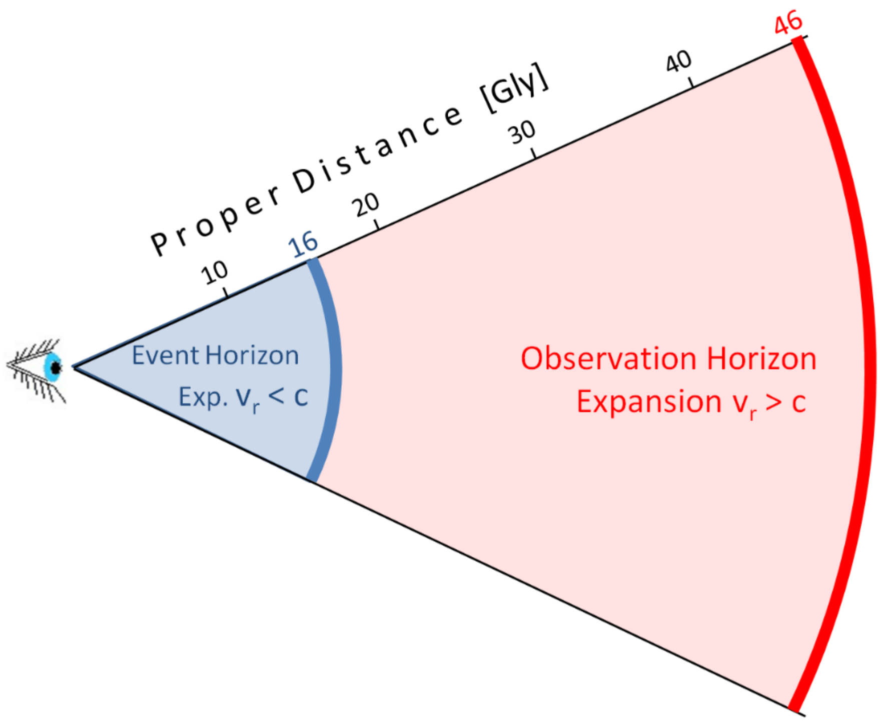
4.2 The Cosmologic Event Horizon or Hubble Radius r_H

The Cosmologic Event Horizon should never be confused with the much larger Observation Horizon. It forms the current limit, beyond which the spatial expansion reaches superluminal velocity, which is not in contradiction to Einstein's ART. The rudimentary approximation of the event horizon is the Hubble radius r_H, which defines the so-called "Hubble Sphere". It is defined by the simple equation {14} of the Hubble-Lemaître law, where the "escape velocity" v_r is simply replaced by the speed of light c:

$$r_H = \frac{c}{H(0)} \qquad \{20\}$$

With a Hubble Constant H(0) of 68 km s^{-1} Mpc^{-1} this simple calculation results in a Hubble Radius r_H of about 14.2 Gly.

The real Event Horizon is based on the cosmological ΛCDM standard model and is, due to the accelerated expansion, with about 16 Gly somewhat larger than the Hubble Radius r_H [1].



4.3 Comparison Observation Horizon versus Event Horizon

As an example, we observe a galaxy at a distance of 30 Gly, i.e. within the red sector on the sketch. This is still within the observation horizon but already clearly outside the Event Horizon. If a supernova lights up in this galaxy at the present time $t = t(0)$, its light will never reach us in the future, because the object is moving away from us at superluminal speed due to the expansion of space. However, the light that we receive from this galaxy today was emitted at a time when the universe was much smaller and the object was still within the former event horizon.

5 The Determination of Cosmological Distances

5.1 The Practical Measurement of the Redshift in the Spectrum

5.1.1 What is measured?

Cosmological distances are measured to galaxies or quasars, located far outside our Milky Way. Professional large telescopes can still record spectra of single stars in directly neighbouring galaxies. However, at greater distances, even such instruments can just record so-called composite- or integrated spectra, composed of billions superposed profiles of individual stars [28]. Whether we obtain here absorption or emission spectra depends mainly on the type and developmental stage of the galaxy, whereas the activity of the region around the central black hole (AGN) plays a decisive role. In the spectral atlas [28] a classification system is presented with typical profiles. As a result of rotation and other effects, the spectral signatures appear generally blurred and sometimes broadened, complicating the measurement of the redshift. With the exception of the directly neighboring Messier galaxies, with amateur telescopes just the bright core can be recorded.

5.1.2 Requirements to the Spectrograph

For an accurate measurement a slit spectrograph is required, which allows the absolute calibration of the spectrum with a calibration light source. However, in [32] the author describes a procedure how to measure large redshifts with a significantly reduced accuracy, even with a slitless transmission grating.

In contrast to the detection of exoplanets, here the redshifts are so large that a resolution of $R \approx 900$ is completely sufficient. This can be achieved, for example, by a spectral grating with 200L mm-1. This enabled, for example, the z-value of the quasar 3C273 to be measured to almost three decimal places. Moreover, the apparent brightness of these objects is so low that even in the professional field high resolutions are out of the question, which would further require correspondingly long exposure times. High-resolution Echelle spectrographs require too much light for this purpose.

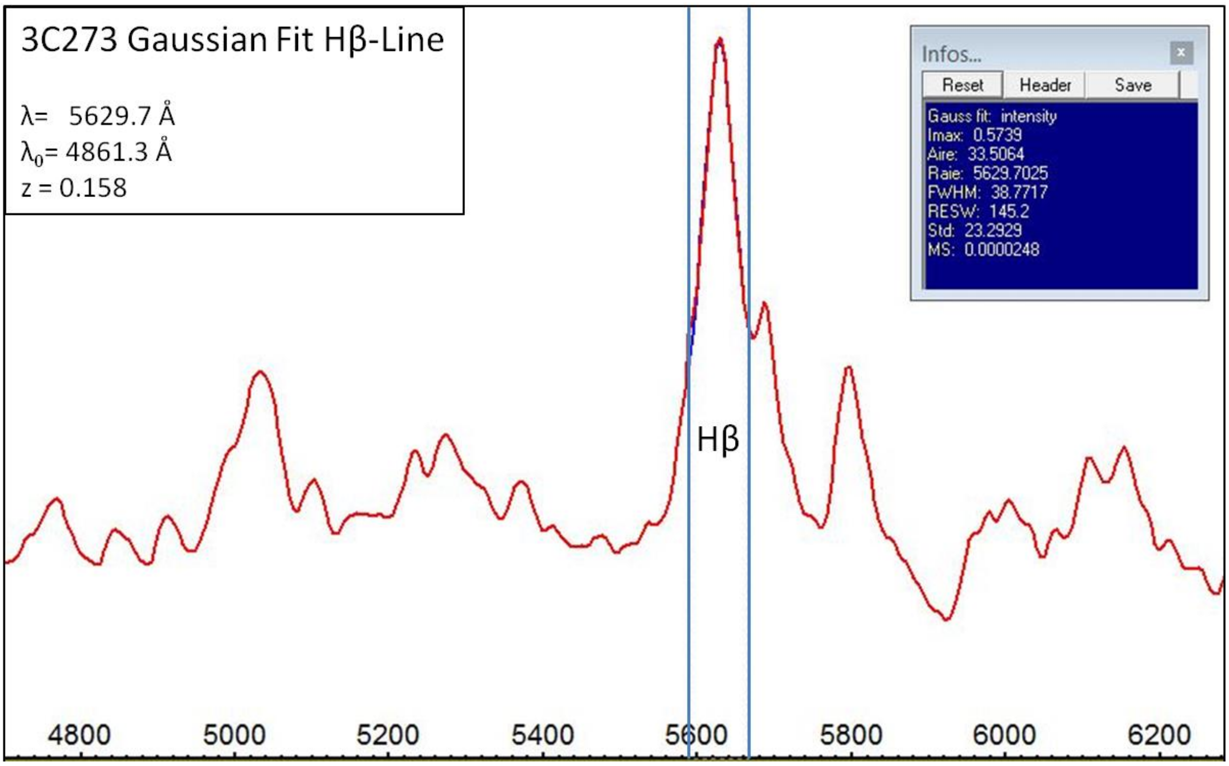
5.1.3 Requirements to the Camera

For spectral recording of these extremely faint objects a cooled astro-camera with a not too dense pixel grid and the binning mode option is required. Today's cameras are mostly optimized for astrophotography, with large, high-resolution sensors and a very narrow pixel grid. For spectroscopy in this case 2x2, or even the 3x3 binning mode should be used to avoid heavy over-sampling and at the same time to increase the sensitivity of the detector. The following spectral image of the quasar 3C273 was recorded with a C8 telescope and the no longer available Atik 314L+ Mono. The Sony sensor ICX285AL has a relatively coarse pixel grid of 1391 x 1039 and large 6.45µm pixels. Nevertheless, already here, to optimize the sampling and the sensor sensitivity, the 2x2 binning mode was necessary. The corresponding possibilities and limitations are described in [27].

5.1.4 Selection of the Spectral Signature to be Measured

Quasars and galaxies with high nuclear activity produce numerous emission lines of the H-Balmer series, as well as of highly ionized metals suitable for redshift measurements. Particularly easy to evaluate are the relatively slim emissions of the Seyfert galaxies. A prime example is M77 with a very bright core [28]. More difficult is the identification of a usable line for pure absorption spectra, like those generated by M31. The Spectral Atlas [28] provides here useful information and examples.

In the case of quasars, these emissions are usually bell-shaped broadened, suggesting to measure the wavelength directly by a Gaussian fit [32]. The following diagram shows the Hβ emission of quasar 3C273, which merges with emissions of other ions to a so-called blend. To optimize the measurement accuracy, the gaussian fit was therefore limited to the upper range of the emission. The Hβ emission was preferred over the Hα line, because the latter at z = 0.158 gets strongly overprinted by the atmospheric Fraunhofer A absorption at ~7'600 Å.



5.1.5 Proportionality of Redshift and Wavelength

Since the redshift according to equation {8} behaves proportional to the wavelength, the z-value remains the same for all evaluated lines, no matter in which range of the spectrum it is measured. However, since the shift Δλ grows with increasing wavelength, a line with the longest possible wavelength should be selected to optimize the measurement accuracy

5.1.6 Heliocentric Correction

At larger redshifts, the measured expansion velocities are so high that a heliocentric correction of the measurement can usually be omitted.

5.1.7 Objects with Strong Redshift

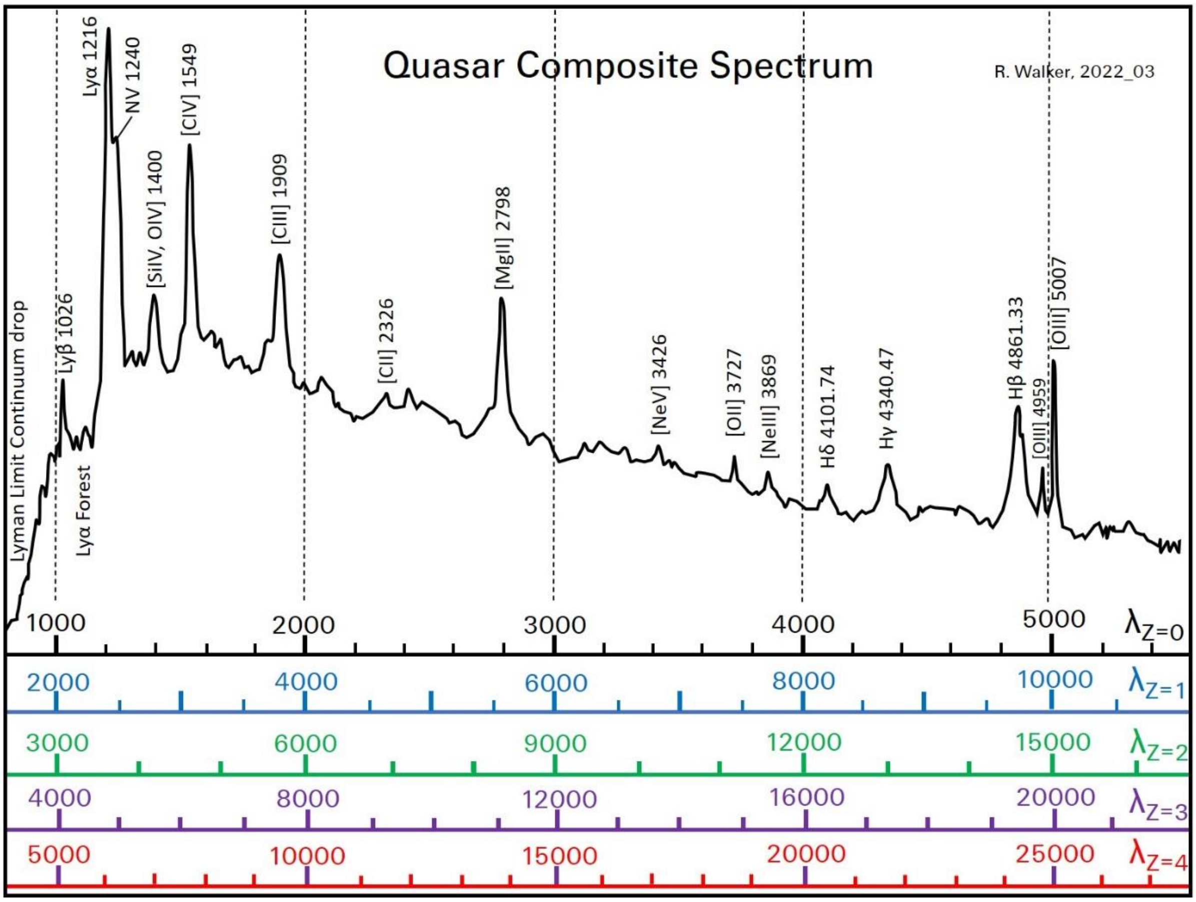
For objects with high z-values, the entire H-Balmer series gets shifted far into the infrared range. Therefore, the Lyα emission of the Lyman series, whose rest wavelengths are located in the UV range, appears in the visual spectrum from about z>2. Here begins also the range of the so-called "High-z Quasars". The following figure shows a quasar composite spectrum, schematically composed according to P. J. Francis et al [18] and D. W. Harris et al [19]. The top scale, labeled in black, shows the associated rest wavelengths λ_0 ranging from 1000 to 5200Å. The lower, colored scales are red-shifted, corresponding to z=1 to z=4 and calculated according to formula {21}.

$$\lambda_z = \lambda_0(1+z) \qquad \{21\}$$

With this graphic the spectral signatures can be determined, which are expected at a certain redshift z in the visual range (approx. 3800 - 8000Å). With increasing z-value, the spectrum recorded in the visual range appears more and more stretched, so that increasingly smaller sections are recorded (example see section 5.1.7).

Here it becomes also evident that at the short wavelength end of the visual range the Lyα emission becomes detectable not before a redshift of $z > 2$. In the range of $z < 2$, mainly the short wavelength Hβ and Hγ lines of the Balmer series, as well as Mg II, CIII and CIV can be applied for the measurement.

The Lyα-emission is mostly the shortest wavelength line, which can still be evaluated for the determination of the redshift. After this follows just the absorptions of the so called "Lyα-Forest" (see section 5.1.7) and possibly a diffuse hump of the Lyβ-line. Beyond the Lyman limit <912 Å we see an abrupt drop of the continuum. From here on, the high-energy UV photons are ionizing the neutral hydrogen of the intergalactic space and thus get absorbed [27].

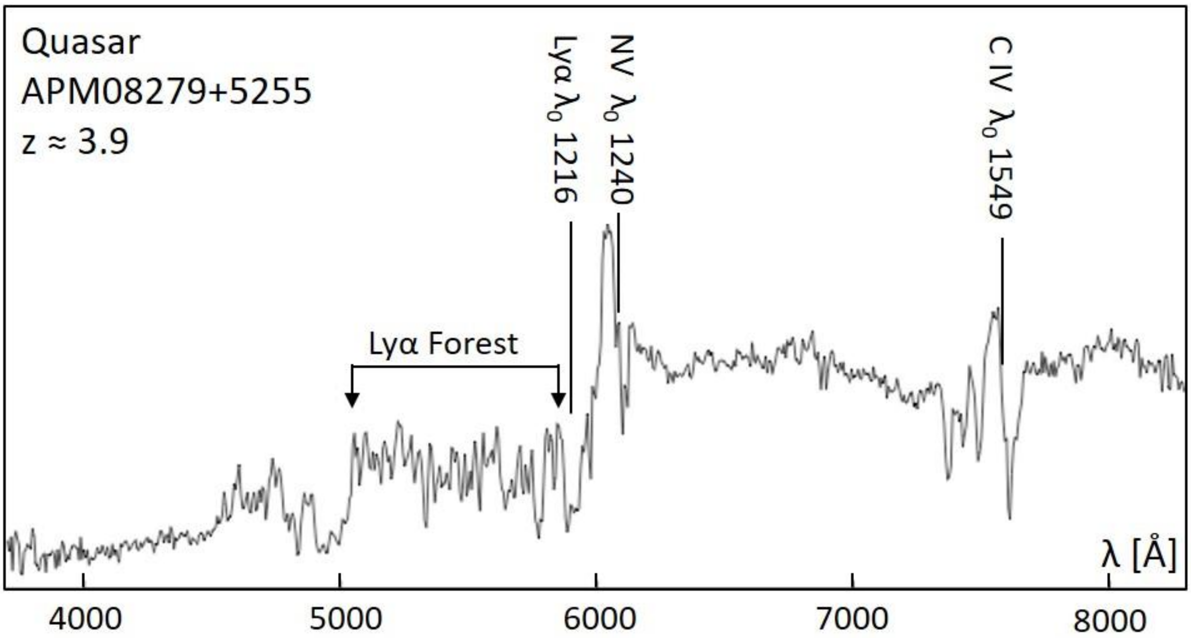


5.1.8 Practical Example Quasar APM08279+5255

The following profile from the NASA/IPAC Extragalactic Database [16] shows the spectrum of the already mentioned quasar APM08279+5255 with a redshift of z ≈ 3.9. The profile is supplemented by the spectral signatures and their rest wavelengths, which are typical for the range around Lyα. Of the Lyman series, as shown here, just the Lyα line can be seen. Probably due to a gas cloud in the vicinity of the quasar [17], it doesn't appear here as emission, but in self-absorption.

Due to the impressive spreading of the spectrum as a result of the high z-value, the highly ionized NV and C IV emissions can be seen here well resolved as inverse (red-shifted) P Cygni profiles [27] - possibly as a result of the contraction processes around the black hole. Therefore, a measurement of the wavelength here must be related not to the slightly blue-shifted peak, but approximately to the central inflection point of the entire P Cygni profile (see the following Fig. and [27].

The sharp absorptions of the "Lyα-Forest" are generated by the interaction of light with intergalactic gas clouds located at different intermediate distances. Further absorptions, e.g. at ~5000 Å, are caused by objects at smaller distances in the foreground, forming this way a complex gravitational lens for this quasar [17].



Note: The NASA/IPAC Extragalactic Database [16] contains recorded spectra of many objects. However, their resolution is mostly low and it is important to note whether rest- or redshifted wavelengths are displayed.

5.1.9 Blazars

Blazars are quasars whose jet, generated by the Supermassive Black Hole, is heading more or less directly towards the solar system. As a result, their spectra in the visual range usually show just a variable continuum without any further signatures. A typical example is Makarian 421, where, by amateur means, no redshift can be measured (profile see Spectral Atlas, Plate 60 [28]).

5.2 The Application of Cosmological Calculation Tools

5.2.1 The Functioning of the Tools

In the context of this script, only methods based on the measured z-value are presented, serving now as the determining variable for the calculation of the different cosmological distances. Today this challenging calculation can be easily done with cosmological calculation tools or "cosmology calculators", which are numerously available in the internet. Their default parameters mostly correspond to the flat ΛCDM model introduced in section 2. For the Hubble constant (by default) mostly $H_0 \approx 68$ km s^{-1} Mpc^{-1} is aplied, i.e. based on measurements of the cosmic background radiation of the Planck Microwave Space Telescope. For all tools the measured z-value is the only distance variable. The time base of these tools is usually the present, i.e. $t = t_0$, and $z = 0$, allowing with the measured z-value to calculate back into the past, i.e. $z > 0$.

The algorithms applied to calculate the cosmological models are mathematically complex and require the numerical integration of differential equations. Between the individual tools, as a result of differently chosen procedures and algorithms, smaller differences may arise. Amateurs are recommended first to start with the default values of the respective model in order to avoid "senseless" results, which for example result in a much too high or too low age of the universe.

5.2.2 Terminology

The terminology used in the individual tools is sometimes different. If in doubt, for a given z-value the results can be compared with other programs. So the Light Travel Time D_T or Lookback Time is sometimes "hidden" behind terms like "Age" or "Age of the Universe". You will never find the "Proper Distance" here, but as already mentioned, it is represented by the term "Comoving Distance" or by "Distance between two redshifts".

Some tools provide a whole range of additional results, such as the Hubble parameter at time t. Others generate diagrams or include a tutorial.

5.2.3 Recommendation

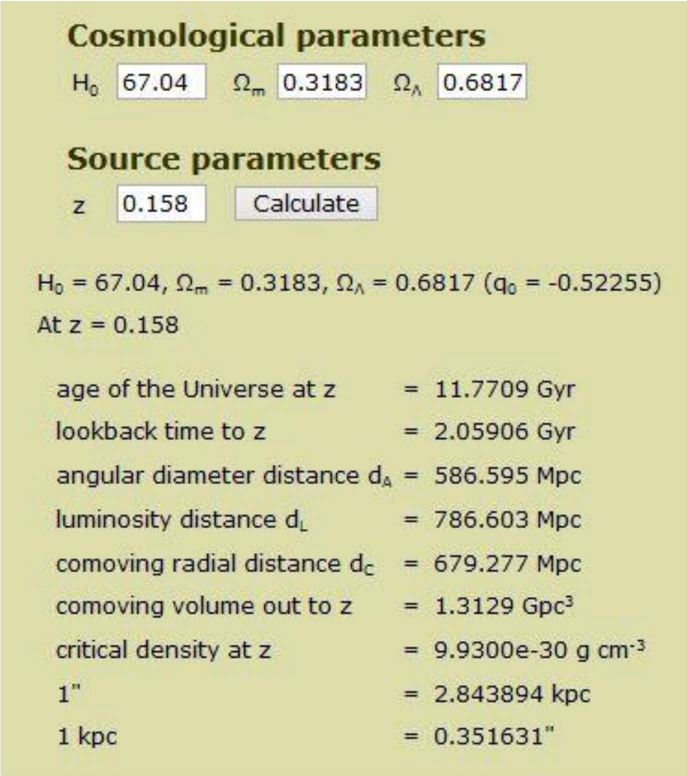
Start with the tools by Ned Wright [24] and Josh Kempner [20] and then compare all tools listed in the bibliography and possibly also others.

5.3 Examples

5.3.1 Quasar 3C273

Measured z-value: z = 0.158

Following Results are according to J. Kempner [20] and N. Gnedin [21] with distances in [Mpc]. For comparison with the Light Travel- or LookbackTime, these distances are converted in the text into [ly] 1 Mpc ≈ 3.26 Mly:



- As expected, the Proper Distance (comoving radial distance dc) with $D_P(t0) = 2.21$ Gly is longer than the Light Travel Time (lookback time to z) of 2.06 Gyr.
- The Luminosity Distance dL is with $D_L(t0) = 2{,}56$ Gly clearly the longest here
- The scale factor a is not provided by the tool here, but can be calculated simply with equation {6} to $a(t) \approx 0.86$, i.e. the universe then had just about 86% of today's size.
- Correspondingly shorter at that time was the distance $D_P(t)$ with just $a\,(t) \cdot D_P(t0) = 0{,}86 \cdot 2.21 = 1{,}90$ Gly. This also corresponds to the current angular diameter distance dA.
- The former value of the Hubble parameter $H(t)$ at time z = 0.158, can additionally be calculated with the tool by N. Gnedin [21]: Based on a Hubble constant of $H(0)$ = 68.14, it results $H(t) = 73.64\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

5.3.2 Quasar APM08279+5255

Measured z-value: z = 3.9

Following Results are according to J. Kempner [20] and N. Gnedin [21] with distances in [Mpc]. For comparison with the Light Travel- or LookbackTime, these distances are converted in the text into [ly] 1 Mpc ≈ 3.26 Mly:

Cosmological parameters

H_0 67.04 Ω_m 0.3183 Ω_Λ 0.6817

Source parameters

z 3.9 Calculate

H_0 = 67.04, Ω_m = 0.3183, Ω_Λ = 0.6817 (q_0 = -0.52255)

At z = 3.9

age of the Universe at z	= 1.5842 Gyr
lookback time to z	= 12.2457 Gyr
angular diameter distance d_A	= 1484.91 Mpc
luminosity distance d_L	= 35652.8 Mpc
comoving radial distance d_C	= 7276.07 Mpc
comoving volume out to z	= 1613.54 Gpc^3
critical density at z	= 3.2197e-28 g cm^{-3}
1"	= 7.199061 kpc
1 kpc	= 0.138907"

- As expected, the difference between the Proper Distance (comoving radial distance dc) of 23.7 Gly and the Light Travel Time (lookback time) of 12.2 Gyr is much larger than in the much closer quasar 3C273.
- The Luminosity Distance dL reaches here with $D_L(t0) = 116$ Gly an "absurd" high value.
- The scale factor a is not provided by the tool here, but can be calculated simply with equation {6} to $a(t) = 0.20$, i.e. the universe then had just about 20% of today's size.
- Correspondingly shorter at that time was the distance $D_P(t)$ with just $a(t) \cdot D_P(t0) = 0.20 \cdot 23.7 = 4.74$ Gly. This also corresponds to the current angular diameter distance dA.
- The former value of the Hubble parameter $H(t)$ at time z = 3.9, can additionally be calculated with the tool by N. Gnedin [21]: Based on a Hubble constant of $H(0)$ = 68.14, it results $H(t) = 411.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

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